DETERMINATION OF THE THERMAL DIFFUSIVITY FROM UNSTEADY TEMPERATURES UNDER HEATING BY LOCAL HEAT SOURCES

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The authors have obtained exact explicit dependences for calculating the thermal diffusivity from measured temperatures of a material when heated by local heat sources of variable power.

The objectives of increasing the information about and simultaneously simplifying the technology of a thermal experiment have motivated improvement of methods of determining the thermophysical characteristics of materials by using the laws of unsteady heat conduction. The present study brings the solution of the problem of determining the thermophysical properties in two-dimensional geometry with heating of semiinfinite specimens by local heat sources of variable power to the development of the approaches used in [1-4].

With reference to the process of propagating heat in an orthotropic material we consider the following mathematical model:

$$\frac{\partial^2 T(r,z,\tau)}{\partial r^2}+\frac{1}{r}\frac{\partial T(r,z,\tau)}{\partial r}-\frac{1}{\eta}\frac{\partial^2 T(r,z,\tau)}{\partial z^2}=\frac{1}{a_r}\frac{\partial T(r,z,\tau,)}{\partial \tau} \tag{1}$$

$$-\lambda_z\frac{\partial T(r,z,\tau)}{dz}\bigg|_{z=0}=q(r,\tau), \tag{2}$$

$$T(r,z,\tau)|_{r\to\infty}=T(r,z,\tau)|_{z\to\infty}=0, \tag{2'}$$

$$T(r,z,0)=0,\ \eta=\lambda_r/\lambda_z, \tag{2''}$$

which occurs, for example, in heating of a half space by local heat sources (distributed or concentrated).

With the help of the integral transforms of Laplace $T(r, z, s) = \int_0^\infty \exp(-s\tau)T(r, z, \tau)d\tau$ and Hankel $T(p, z, s) = \int_0^\infty rJ_0(pr)T(r, z, \tau)dr$ the solution of the system (1)-(2") can be re-represented in the form

$$T(r,z,s)=\frac{1}{\sqrt{\lambda_r\lambda_z}}\int_0^\infty\frac{pJ_0(pr)q(p,s)\exp\left[-z\left[\sqrt{\eta\left(p^2+\frac{s}{a_r}\right)}\right]\right]}{\sqrt{p^2+\frac{s}{a_r}}}dp, \tag{3}$$

where $q(p, s) = \int_0^\infty rJ_0(pr)q(r, s)dr$. For heat sources concentrated in a neighborhood of radius R_0, uniformly distributed within a neighborhood of radius R_0, and concentrated at a point, respectively, the following expressions occur for $q(p, s)$: $\frac{Q(s)}{2\pi}J_0(pR_0)$, $\frac{Q(s)}{\pi R_0 p}J_1(pR_0)$, $\frac{Q(s)}{2\pi}$, where $Q(s)$ is the power of the heat source.

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TABLE 1. Results of Estimates of Parameters a_r and η from "Exact" Original Data

τ_k, sec	η_1 from T_1, T_2; T_1, T_3	η_2 from T_1, T_3; T_2, T_3	$a_r \cdot 10^6$, m²/sec		$a_r \cdot 10^6$, m²/sec with $\eta=1$		
			by T_1, T_2; η_1	by T_1, T_3; η_2	by T_1, T_2	by T_1, T_3	by T_2, T_3
0,4	−6,19	10,20	$8{,}6 \cdot 10^6$	$2{,}0 \cdot 10^6$	$1{,}2 \cdot 10^4$	$4{,}2 \cdot 10^3$	0
0,8	8,00	8,29	34,20	36,23	0,60	0,78	0,86
1,2	4,29	4,67	10,85	15,59	0,63	0,72	0,76
1,6	3,85	3,95	9,31	9,73	0,66	0,75	0,79
2,0	3,81	3,84	9,44	9,58	0,69	0,78	0,81
2,4	3,83	3,84	9,74	9,79	0,70	0,79	0,83
2,8	3,86	3,86	10,02	10,03	0,71	0,81	0,84
3,2	3,89	3,88	10,28	10,26	0,72	0,82	0,85
3,6	3,92	3,91	10,49	10,47	0,72	0,82	0,86
4,0	3,95	3,94	10,67	10,64	0,72	0,83	0,86
4,4	3,96	3,96	10,77	10,76	0,73	0,83	0,87
4,8	3,97	3,97	10,84	10,83	0,73	0,83	0,87

Allowing for Eqs. (5) and (3) the connection between the transforms of the temperatures $T(0, z_1, s)$ and $T(0, z_2, s)$ in the case of a source concentrated in a neighborhood of radius R_0 can be written in the form

$$\varphi(s) = \frac{T(0, z_1, s)}{T(0, z_2, s)} = \sqrt{\frac{\eta z_2^2 + R_0^2}{\eta z_1^2 + R_0^2}} \exp\left[-\sqrt{\frac{s}{a_r}}\left(\sqrt{\eta z_1^2 + R_0^2} - \sqrt{\eta z_2^2 + R_0^2}\right)\right], \tag{4}$$

whence we have

$$\frac{d\varphi}{ds} = -\frac{\sqrt{\eta z_1^2 + R_0^2} - \sqrt{\eta z_2^2 + R_0^2}}{2\sqrt{s a_r}}\varphi. \tag{5}$$

After transferring to the originals, in accordance with [6] we obtain

$$a_r = \frac{1}{4}[(\eta z_1^2 + R_0^2)^{1/2} - (\eta z_2^2 + R_0^2)^{1/2}]^2 \left[\frac{\psi_1(\tau)}{\psi_2(\tau)}\right]^2, \tag{6}$$

where

$$\psi_1(\tau) = \frac{1}{\sqrt{\pi}} \int_0^\tau T_1(\tau - \theta) \int_0^\theta \frac{T_2(\xi)}{\sqrt{\theta - \xi}} d\xi d\theta; \tag{6'}$$

$$\psi_2(\tau) = \int_0^\tau (\tau - 2\theta) T_1(\theta) T_2(\tau - \theta)\, d\theta; \tag{6''}$$

$$T_1(\theta) \equiv T(0, z_1, \theta),\ T_2(\theta) \equiv T(0, z_2, \theta).$$

Equation (6) relates the parameters a_r and η explicitly with the measured temperatures at two points on the symmetry axis, for arbitrary laws of time variation of the power $q(\tau)$ of a heat source concentrated in the neighborhood R_0.

In the case when $\eta = 1$ from Eq. (6) we can compute a set of values of the parameter $a \equiv a_r$ for any previously given time intervals τ_i during the time t_p, we can estimate the errors in determining a desired parameter and can check the correctness of representing the real situation by a linear model (condition of constancy of the thermophysical characteristics).

When $\eta \neq 1$, the value of the parameter η can be obtained if we know the measured temperatures at some point with coordinates $r = 0$, $z = z_3$. In this case from Eq. (6) we have that

$$\frac{(\eta z_i^2 + R_0^2)^{1/2} - (\eta z_j^2 + R_0^2)^{1/2}}{(\eta z_i^2 + R_0^2)^{1/2} - (\eta z_k^2 + R_0)^{1/2}} = \frac{\psi_1[T_i, T_k]\,\psi_2[T_i, T_j]}{\psi_2[T_i, T_k]\,\psi_1[T_i, T_j]} \equiv F_\nu, \tag{7}$$

$$i, j, k = 1, 2, 3,\ \nu = 1, 2, 3,$$

TABLE 2. Results of Estimates of Parameters a_r and η in the Presence of Random Errors in the Original Data

Parameter	τ, sec										
	0,4	0,8	1,2	1,6	2,0	2,4	2,8	3,2	3,6	4,0	4,4
η_1 by T_1, T_2; T_1, T_3	—6,2	8,0	4,3	3,9	3,8	3,8	3,85	3,9	3,9	3,95	3,95
	—6,2	6,2	4,4	3,7	3,9	5,4	5,0	4,6	4,5	4,5	4,5
	—6,2	8,4	5,5	3,6	3,6	4,8	4,8	4,1	3,8	3,5	3,1
	—6,2	7,8	4,3	4,2	3,8	2,7	2,9	3,4	3,8	3,7	3,5
	—6,2	7,3	4,6	2,3	2,5	3,3	3,5	3,4	3,1	3,1	3,2
η_2 by T_1, T_3; T_2, T_3	10,2	8,3	4,7	3,9	3,8	3,8	3,85	3,9	3,9	3,95	3,95
	10,2	7,5	3,6	4,4	3,5	4,8	5,3	4,9	4,6	4,5	4,5
	10,5	8,5	5,6	4,0	3,5	4,3	4,9	4,5	4,1	3,8	3,4
	10,5	8,3	4,3	4,0	4,15	3,2	2,8	3,1	3,5	3,7	3,6
	10,0	8,8	5,4	3,0	2,2	2,9	3,4	3,5	3,3	3,1	3,1
$a_r \cdot 10^5$, m²/sec by T_1, T_2; η_1	$5 \cdot 10^4$	3,42	1,09	0,93	0,94	0,97	1,00	1,02	1,05	1,07	1,08
	$6 \cdot 10^4$	2,22	1,12	0,88	1,00	1,78	1,62	1,36	1,33	1,32	1,35
	$6 \cdot 10^4$	3,71	1,66	0,81	0,88	1,48	1,50	1,15	0,98	0,84	0,67
	$6 \cdot 10^4$	3,35	1,07	1,07	0,92	0,53	0,60	0,78	0,97	0,94	0,86
	$5 \cdot 10^4$	2,92	1,24	0,36	0,43	0,73	0,85	0,80	0,70	0,68	0,73
$a_r \cdot 10^5$, m²/sec by T_1, T_3; η_2	$3 \cdot 10^4$	3,62	1,26	0,97	0,96	0,98	1,00	1,03	1,05	1,06	1,08
	$3 \cdot 10^4$	3,08	0,78	1,17	0,82	1,48	1,74	1,51	1,39	1,35	1,34
	$3 \cdot 10^4$	3,81	1,74	0,99	0,83	1,23	1,54	1,34	1,14	0,97	0,80
	$3 \cdot 10^4$	3,71	1,09	0,98	1,08	0,71	0,58	0,70	0,85	0,94	0,91
	$3 \cdot 10^4$	3,96	1,61	0,58	0,35	0,60	0,81	0,85	0,77	0,70	0,71

where $\psi_1[T_i, T_k]$, $\psi_2[T_i, T_k]$, $\psi_1[T_i, T_j]$, $\psi_2[T_i, T_j]$ denote integral combinations of $\psi_1(\tau)$, $\psi_2(\tau)$, computed with the aid of Eqs. (6')-(6") according to the appropriate pairs of T_i, T_j and T_k.

From Eq. (7) using identical transformations we can obtain explicit expressions for estimating the parameter η:

$$\eta = \frac{4(1-F_\nu)F_\nu[\bar{z}_i^2(1-F_\nu) - \bar{z}_j^2 + F_\nu \bar{z}_k^2]}{[\bar{z}_i^2(1-F_\nu)^2 + \bar{z}_j^2 - F_\nu \bar{z}_k^2]^2 - 4(1-F_\nu)^2 \bar{z}_j^2 \bar{z}_k^2} \tag{8}$$

or

$$\eta = \frac{4F_\nu[\bar{z}_i^2(1-F_\nu)^2 - \bar{z}_j^2 - \bar{z}_k^2 F_\nu^2 + F_\nu(\bar{z}_j^2 + \bar{z}_k^2)]}{[\bar{z}_i^2(1-F_\nu)^2 - \bar{z}_j^2 - \bar{z}_k^2 F_\nu^2] - 4F_\nu^2 \bar{z}_j^2 \bar{z}_k^2}, \tag{8'}$$

$$\bar{z}_i = z_i/R_0, \quad \bar{z}_j = z_j/R_0, \quad \bar{z}_k = z_k/R_0.$$

Thus, from the known pairs of values of T_i, T_j, T_k, using Eq. (8) or (8') we determine the parameter η for any previously given number of time intervals in the duration of the test t_p. The set of estimates of the parameter η obtained can be used to analyze the convergence of the computed values to some constant value and to check the correctness of representing the real situation by the linear model.

We note that estimates of the parameter η can be used for various F_ν computed according to different combinations of pairs from T_i, T_j and T_k. There are evidently two such independent estimates in this case. After the parameter η is determined the estimate of thermal diffusivity a_r can be used in Eq. (6) along with any combination of the available T_i, T_j, T_k.

In the case of a local source concentrated at the origin of coordinates (a point source), using the above integral transformations we can write the relation in Laplace transform space between the temperatures $T(r_1, z_1, s)$ and $T(r_2, z_2, s)$ in the form

$$\frac{T(r_1, z_1, s)}{T(r_2, z_2, s)} = \frac{(\eta z_2^2 + r_2^2)^{1/2}}{(\eta z_1^2 + r_1^2)^{1/2}} \exp\left[-\sqrt{\frac{s}{a_r}}\left(\sqrt{\eta z_1^2 + r_1^2} - \sqrt{\eta z_2^2 + r_2^2}\right)\right], \tag{9}$$

where r_1, z_1, r_2, z_2 are coordinates of the points at which the temperatures were measured. From Eq. (9), in analogy with the case examined above, we obtain

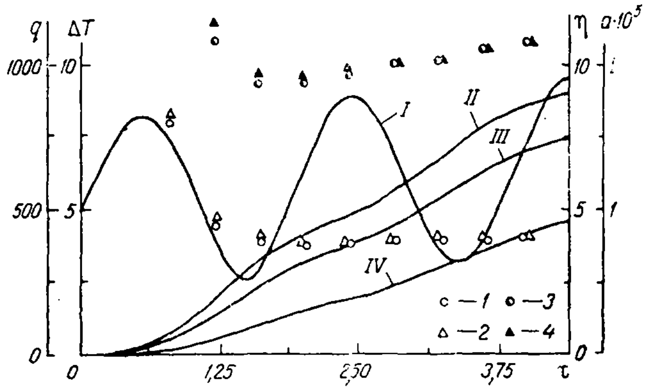

Fig. 1. Original data for computing and results of estimates of the parameters η and a_r: I) source power; II, III, IV) temperatures on the axis of symmetry (r = 0) at distances 0, 2 and 4 mm from the surface, respectively; 1) η_1 from T_1, T_2; T_1, T_3; 2) η_2 from T_1, T_3; T_2, T_3; 3) a_r from T_1, T_2, η_1; 4) a_r from T_1, T_3, η_2. q, W/m; ΔT, K; a_r, m²/sec; τ, sec.

$$a_r = \frac{1}{4}\,[(\eta z_1^2 + r_1^2)^{1/2} - (\eta z_2^2 + r_2^2)^{1/2}]^2 \left[\frac{\tilde{\psi}_1(\tau)}{\tilde{\psi}_2(\tau)}\right]^2, \tag{10}$$

where $\tilde{\psi}_1$, $\tilde{\psi}_2$ are computed from Eqs. (6') and (6") using as $T_1(\theta)$ and $T_2(\theta)$ the quantities $T(r_1, z_1, \theta)$ and $T(r_2, z_2, \theta)$, respectively.

If $\eta = 1$, from Eq. (10) we can estimate the parameter $a \equiv a_r$ for the chosen number of time intervals in the test. But if there is a basis for putting $\eta \neq 1$, then to determine η we can use Eq. (8), which postulates the presence of measured temperatures at three points with coordinates z_1, z_2, z_3 and $r_1 = r_2 = r_3 = 0$. But if the coordinates of the measured temperature points are arbitrary, then the parameter η can be determined from the relation

$$\frac{(\eta z_1^2 + r_1^2)^{1/2} - (\eta z_2^2 + r_2^2)^{1/2}}{(\eta z_1^2 + r_1^2)^{1/2} - (\eta z_3^2 + r_3^2)^{1/2}} = \frac{\tilde{\psi}_1[T(r_1, z_1, \tau), T(r_3, z_3, \tau)]\,\tilde{\psi}_2[T(r_1, z_1, \tau), T(r_2, z_2, \tau)]}{\tilde{\psi}_1[T(r_1, z_1, \tau), T(r_2, z_2, \tau)]\,\tilde{\psi}_2[T(r_1, z_1, \tau), T(r_3, z_3, \tau)]}, \tag{11}$$

which in general reduces to an equation of second order in the parameter η.

When the half space surface is heated by a distributed heat source in the form of a disk of radius R_0, in analogy with the cases examined above we can obtain a relation of the form

$$\frac{T(0, z_1, s)}{T(0, z_2, s)} = \frac{\exp\left[-\sqrt{\frac{s}{a_r}}\,\eta^{1/2} z_1\right] - \exp\left[-\sqrt{\frac{s}{a_r}}\,(\eta z_1^2 + R_0^2)^{1/2}\right]}{\exp\left[-\sqrt{\frac{s}{a_r}}\,\eta^{1/2} z_2\right] - \exp\left[-\sqrt{\frac{s}{a_r}}\,(\eta z_2^2 + R_0^2)^{1/2}\right]}, \tag{12}$$

which relates the transforms of temperatures at the points z_1 and z_2 on the symmetry axis (r = 0).

If clearly $\eta = 1$, then from Eq. (12) we can obtain comparatively simple relations to determine the thermal diffusivity for certain $z_2 = f(z_1)$. For example, if $z_2 = 1.25 z_1 + 0.75(z_1^2 + R_0^2)^{1/2}$, then

$$\varphi(s) = \exp\left[\sqrt{\frac{s}{a}}\,(z_2 - z_1)\right]\left\{1 + \exp\left[-\sqrt{\frac{s}{a}}\,(\sqrt{z_2^2 + R_0^2} - z_2)\right]\right\},\quad a \equiv a_r, \tag{13}$$

where $\varphi(s) = T(0, z_1, s)/T(0, z_2, s)$. Differentiating Eq. (13) with respect to s, we have

$$[\varphi'(s)\sqrt{s}]' + \frac{(z_2^2 + R_0^2)^{1/2} - (3z_2 - 2z_1)}{2\sqrt{a}}\,\varphi'(s) = \frac{(z_2 - z_1)\,[(z_2^2 + R_0^2)^{1/2} - (2z_2 - z_1)]}{4a\sqrt{s}}\,\varphi(s). \tag{14}$$

After carrying out the differentiation in Eq. (14) and a number of identical transformations and transferring to the originals, we obtain a relation for determining the parameter a:

$$\psi_1(\tau)\,a+\sqrt{a}\,\frac{(z_2^2+R_0^2)^{1/2}-(3z_2-2z_1)}{2}\,\psi_2(\tau)-\frac{(z_2-z_1)}{4}\,[(z_2^2+R_0^2)^{1/2}-(2z_2-z_1)]\,\psi_3(\tau)=0, \tag{15}$$

where

$$\psi_1(\tau)=\int\limits_0^\tau(3\theta-\tau)\,T_2(\theta)\,f_1(\tau-\theta)\,d\theta-\frac{3}{2}\int\limits_0^\tau T_2(\theta)f(\tau-\theta)\,d\theta; \tag{15'}$$

$$f_1(\theta)=\int\limits_0^\theta(\theta-2\xi)\,T_2(\theta-\xi)\,T_1'(\xi)\,d\xi;\quad f(\theta)=\int\limits_0^\theta(\theta-2\xi)\,T_1(\xi)\,T_2(\theta-\xi)\,d\xi;$$

$$\psi_2(\tau)=\frac{1}{\sqrt{\pi}}\int\limits_0^\tau f_2(\theta)\,[f_1(\tau-\theta)-f(\tau-\theta)]\,d\theta; \tag{15''}$$

$$f_2(\theta)=\int\limits_0^\theta\frac{T_2(\xi)}{\sqrt{\theta-\xi}}\,d\xi;\quad f_3(\theta)=\int\limits_0^\theta T_1(\xi)\,T_2(\theta-\xi)\,d\xi;$$

$$\psi_3(\tau)=\int\limits_0^\tau f_3(\theta)\,T_2(\tau-\theta)\,d\theta; \tag{15'''}$$

$$T_1(\theta)\equiv T(0,\,z_1,\,\theta);\quad T_2(\theta)\equiv T(0,\,z_2,\,\theta).$$

If $\eta\neq 1$, for this variant we were not able to obtain rather simple explicit relations to determine the parameters a_r and η from the measured temperatures in a single test.

Besides the relations presented, comparatively simple relations to determine a_r and η can be obtained when we are using the results of temperatures measured in several tests. For example, if there are tests with local sources concentrated in neighborhoods of radii R_{01} and R_{02}, then with the condition $q_1(\tau)=q_2(\tau)$ from Eq. (3) for the point $r=0$, $z=z_1$ we obtain

$$a_r=\frac{1}{4}\,[\sqrt{\eta z_1^2+R_{01}^2}-\sqrt{\eta z_1^2+R_{02}^2}]^2\left[\frac{\psi_1(\tau)}{\psi_2(\tau)}\right]^2, \tag{16}$$

where ψ_1, ψ_2 are calculated from (6')-(6"), and for T_1 and T_2 we use the results of measurements of temperatures at the point $r=0$, $z=z_1$ in the first and second tests, respectively.

If $\eta\neq 1$. then to determine a_r and η when measuring temperatures at only one point ($r=0$, $z=z_1$) we can use the experimental data obtained in one test with a source concentrated in a neighborhood of radius R_{03}, and here it is postulated that $q_1(\tau)=q_2(\tau)=q_3(\tau)$. In the latter case to determine the parameter η a formula of the type of Eq. (8') is valid:

$$\eta^*=\frac{4(1-F_\nu)F_\nu[\bar{z}_i^2(1-F_\nu)-\bar{z}_j^2+F_\nu\bar{z}_k^2]}{[\bar{z}_i^2(1-F_\nu)^2+\bar{z}_j^2-F_\nu\bar{z}_k^2]^2-4(1-F_\nu)^2\bar{z}_j^2\bar{z}_k^2}, \tag{17}$$

where $\eta^*=\eta^{-1}$, where $\bar{z}_i=R_{0i}/z_i$, $\bar{z}_j=R_{0j}/z_j$, $\bar{z}_k=R_{0k}/z_k$, $i,\ j,\ k=1,\ 2,\ 3$.

Comparatively simple explicit expressions to determine a_r and η can be obtained when using the measured temperatures in several tests with different sizes of circular sources distributed within a neighborhood). For example, having available data of two tests with source dimensions R_1 and R_2, with the condition $q_1(\tau)\equiv q_2(\tau)$ on the line $r=0$ for $z=z_1$, $z=z_2$ we have

$$\frac{T_1(0,\,z_1,\,s)}{T_2(0,\,z_2,\,s)}=\frac{\exp\left[-\sqrt{\frac{s}{a_r}}\,\eta^{1/2}z_1\right]-\exp\left[-\sqrt{\frac{s}{a_r}}\,(\eta z_1^2+R_1^2)^{1/2}\right]}{\exp\left[-\sqrt{\frac{s}{a_r}}\,\eta^{1/2}z_2\right]-\exp\left[-\sqrt{\frac{s}{a_r}}\,(\eta z_2^2+R_2^2)^{1/2}\right]}. \tag{18}$$

Evidently, if $z_1=2z_2$, $R_1=2R_2$, then

$$\varphi(s) = \exp\left[-z_2\eta^{1/2}\sqrt{\frac{s}{a_r}}\right]\left\{1+\exp\left[-\sqrt{\frac{s}{a_r}}\left(\sqrt{\eta z_2^2+R_2^2}-\eta^{1/2}z_2\right)\right]\right\},$$
$$\varphi(s) \equiv T_1(0,\ z_1,\ s)/T_2(0,\ z_2,\ s). \tag{19}$$

For $z_1 = z_2 = 0$ it follows from Eq. (19) that

$$\varphi'(s) = -\frac{R_2}{2\sqrt{sa_r}}\varphi(s) + \frac{R_2}{2\sqrt{sa_r}}, \tag{20}$$

whence, after differentiating and transferring to the originals, we obtain

$$a_r = \frac{R_2^2}{4}\left[\frac{\bar{\psi}_1(\tau)}{\bar{\psi}_2(\tau)}\right]^2, \tag{21}$$

where

$$\bar{\psi}_1(\tau) = \frac{1}{\sqrt{\pi}}\int_0^\tau T_2(\tau-\theta)\int_0^\theta [T_2(\xi)-T_1(\xi)](\theta-\xi)^{-1/2}\,d\xi d\theta; \tag{21'}$$

$$\bar{\psi}_2(\tau) = \int_0^\tau (\tau-2\theta)T_1(\theta)T_2(\tau-\theta)\,d\theta. \tag{21''}$$

And if $z_1,\ z_2 \neq 0$, then

$$(\varphi'\sqrt{s})' + \frac{1}{2}[(\eta z_2^2+R_2^2)^{1/2}+\eta^{1/2}z_2]a_r^{-1/2}\varphi' + \frac{1}{4}(\eta z_2^2+R_2^2)^{1/2}z_2\eta^{1/2}a_r^{-1}s^{-1/2} = 0, \tag{22}$$

and after transferring to the originals we have

$$\bar{\psi}_1(\tau)a_r + \frac{1}{2}[(\eta z_2^2+R_2^2)^{1/2}+\eta^{1/2}z_2]a_r^{1/2}\bar{\psi}_2(\tau) + \frac{1}{4}(\eta z_2^2+R_2^2)^{1/2}z_2\eta^{1/2}\bar{\psi}_3(\tau) = 0, \tag{23}$$

where $\bar{\psi}_1$, $\bar{\psi}_2$, $\bar{\psi}_3$ are computed from Eqs. (15')-(15'''), and $T_1 \equiv T_1(0,\ z_1,\ \tau)$, $T_2 \equiv T_2(0,\ z_2,\ \tau)$. When $\eta \neq 1$ we can use Eq. (23) along with Eqs. (21) and (15) to determine the thermal diffusivity. But if the parameter η is unknown, we can proceed differently, depending on the volume of measured information. From Eq. (23) on the basis of the postulate that a_r and η are constant it follows that

$$\frac{(\eta z_2^2+R_2^2)^{1/2}+\eta^{1/2}z_2}{2\sqrt{a_r}} = \frac{\psi_{1i}[1-(\psi_{1j}\psi_{3i})/(\psi_{1i}\psi_{3j})]}{\psi_{2i}[(\psi_{2j}\psi_{3i})/(\psi_{2i}\psi_{3j})-1]} \equiv p_1, \tag{24}$$

$$\frac{(\eta z_2^2+R_2^2)^{1/2}\eta^{1/2}z_2}{4a_r} = \frac{\psi_{1i}[1-(\psi_{1j}\psi_{2i})/(\psi_{1i}\psi_{2j})]}{\psi_{3i}[(\psi_{3j}\psi_{2i})/(\psi_{3i}\psi_{2j})-1]} \equiv p_2, \tag{25}$$

where the subscripts i and j refer to different time intervals of the two tests (with sources of radius R_1 and R_2 and identical $q(\tau)$), or to arbitrary time intervals of two series of tests with different $q(\tau)$ in each series. It follows from Eqs. (24) and (25) that

$$\eta = \frac{R_2^2}{2z_2^2}\left(\sqrt{\frac{\beta-2}{\beta(\beta-4)}}-\right);\ \beta = p_1^2p_2. \tag{26}$$

After η is determined from Eq. (26) we can find a_r from Eq. (23).

If, besides the measured temperatures in two tests with source radii R_1 and R_2 (at points $z = z_1$ in the first first and $z = z_2$ in the second), in these we measure temperatures at the points $r = 0$, $z = 0$, then the parameter a_r is easily determined from Eq. (21), and the parameter η is determined directly from Eq. (23), which can be expressed relative to η.

Some of the relations obtained were confirmed by formulating a number of model problems, where as test data we used computed temperatures of a half space with heating by local sources of variable power. The data from one such numerical experiment are shown in Fig. 1 and in Tables 1 and 2. In this experiment the radius of the local source concentrated in a neighborhood was 0.01 m, and the parameters λ_r, λ_z, a_r, η were: $\lambda_r = 40\ W\cdot m^{-1}\cdot K^{-1}$, $\lambda_z = 10\ W\cdot m^{-1}\cdot K^{-1}$, $a_r = 0.111\cdot 10^{-4}\ m^2\cdot sec^{-1}$, $\eta = 4$; in the data reduction we used temperatures computed for the points with coordinates $r = 0$; $z_1 = 0$; $z_2 = 2$ mm; $z_3 = 4$ mm; the time step was 0.1 sec.

Estimates of the parameters a_r and η were generated both as "exact" original data and as data containing random errors with a uniform distribution and $\sigma_T = 0.005|T|$. From the results shown in Table 1 it follows that estimates of the parameter η using different combinations of T_i, T_j, T_k under the condition $\sigma_T = 0$ converge quite rapidly to the true value, equal to 4. The values of η (η_1, η_2) obtained were used to compute two sets of estimates of the parameter a_r. The values of the parameter a_r appearing in both sets of estimates show very good interagreement and converge rather rapidly to the true values with increasing time of the test intervals used in the processing. Table 1 also shows estimates of the parameter a_r obtained under the assumption $\eta = 1$. As follows from the results shown in this case the computed values of a_r differ appreciably from the true value, and in addition one observes a noticeable difference of the estimates obtained when different combinations of T_i, T_j, T_k are used.

The influence of interference contained in the experimental information is illustrated by the data of Table 2, where the first columns for each parameter contain estimates from the "exact" original data, and the remaining columns have data from different tests with $\sigma_T = 0.005|T|$. As can be seen from the results shown, the presence of random errors of "average" level allows us to obtain estimates of acceptable accuracy for the unkown parameters η and a_r even for comparatively short tests, which is due to some extent to the filtering properties of the integration process performed in the computed dependences. Naturally, in each specific case the level of reliability of the estimates obtained is determined by the behavior of the corridor of errors of the computed values of the desired parameters as a function of the length of the time intervals of the tests used to process them. This corridor in turn can be constructed by using simple algorithms to compute elements of the error matrix of the parameters estimated.

NOTATION

T, temperature; q, heat source power; r, z, spatial coordinates; τ, θ, ξ, time; a, a_r, thermal diffusivity; λ_r, λ_z, thermal conductivity in the directions r and z; σ, rms error of the measured temperatures; J_ν, Bessel function of order ν.

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